Summary

We first review some of the definitions of brittleness index (BI) that have been used in the recent literature concerning oil and gas exploration and production from low porosity, low permeability rocks. We will then argue that the definitions characterizing the BI of rocks either by their elastic properties, by their mineralogical composition or by their strength characteristics, are all equivalent and typically result in a higher BI assigned to quartz-rich rocks than to clay-rich lithologies. Therefore the majorities of recent definitions of BI are simply a rock-type indicator and are useful as such. However, the separation of rocks into brittle/ductile lithologies on the basis of a calculated BI is not necessarily an indicator of brittle or ductile failure during hydraulic stimulation. We therefore propose that brittleness index is potentially an unfortunate choice of words and can, at worst, be misleading. We will then show how incorporation of elastic and strength properties into a geomechanical model, which additionally includes the stress state and pore pressure, can be used to determine (i) whether hydraulic fractures are likely to be contained in the resource layer and or will grow out of zone (ii) whether rock fails predominantly in tension or in shear during hydraulic stimulation, and (iii) whether hydraulic stimulation will predominantly create new fractures or is likely to re-activate pre-existing fractures and other planes of weakness (such as bedding boundaries).

Some definitions of brittleness index in the recent literature

There are three predominant groups of definitions for brittleness index (BI) in the recent literature of unconventional resource exploration and production. The three groups characterize the BI of rocks from their elastic properties, their petrophysical properties and their strength properties, respectively. The fact that there is not a single definition of a brittleness index, but a confusing amount of different brittleness indices is easily verified by simple internet search, and has been highlighted in discussions, e.g. Hall (2013).

The first definition, predominant in the geophysical literature, states that rocks characterized by a high brittleness index are defined by a high Young’s modulus (E) and low Poisson’s ratio (ν). This definition seems to trace back to a SPE paper by Rickman et al. (2008). Brittleness index is then simply a suitable combination of any two elastic moduli. It is then also immaterial whether the two elastic moduli used to define a BI are Young’s modulus (E) and Poissons ratio (ν), as in the case of Rickman et al., (2008), or a specific combination of Lamé parameters λ and μ, as used by Goodway et al. (2010), or any other suitable combination of Vp, Vs and bulk density.

The second class of definitions of brittleness index is based on mineral content of rocks. For example, Jarvie et al. (2007) use the following definition:

$$ BI = \frac{V_{\text{Quartz}}}{V_{\text{Quartz}} + V_{\text{Calcite}} + V_{\text{Clay}}} $$

Eq. 1

and Wang and Gale (2009) propose...
Brittleness index

In this next section, we will make the argument that all three proffered definitions for a brittleness index are equivalent, and are each essentially a lithology indicator. A high brittleness index for each definition indicates a sandstone (or quartz rich lithology), and a low brittleness index indicates a shale (or clay rich lithology).

Looking at the mineralogical definition of brittleness index, we pointed out that the brittleness index is a measure of the volume fraction of stiff mineral (such as Quartz or Quartz + Dolomite) as part of the entire matrix volume. A higher proportion of stiff minerals cause an aggregate material to have a higher Young’s modulus than an aggregate material consisting of components of soft minerals. For example, derivation of Young’s moduli from Bulk moduli and Poisson’s ratios tabulated in Mavko, Mikerji and Dvorkin (2009) gives $E_{\text{Quartz}} = 93.0 \text{GPa}$, $E_{\text{Calcite}} = 83.0 \text{GPa}$, $E_{\text{Dolomite}} = 114.0 \text{GPa}$, $E_{\text{Kaolinite}} = 3.2 \text{GPa}$, $E_{\text{Gulf Clays}}(\text{Han}) = 24.0 \text{GPa}$ and $E_{\text{Quartz with clay}} = 77.0 \text{GPa}$. Young’s moduli for different clay minerals can also be calculated from the data given in Wang, Wang and Cates (1998), where Young’s moduli range between 16.5 – 71.0 GPa for clay minerals including montmorillonite, kaolinite, illite and smectite. Quartz tends to be a stiffer mineral than clay minerals and in an aggregate material, containing a combination of minerals, the moduli will be a weighted sum of the individual moduli. Similarly, Poisson’s ratios for different minerals making up the matrix in rocks under consideration in unconventional resource plays range from $\nu_{\text{Quartz}} = 0.08$, $\nu_{\text{Calcite}} = 0.32$, $\nu_{\text{Dolomite}} = 0.3$, $\nu_{\text{Kaolinite}} = 0.14$, $\nu_{\text{Gulf Clays}}(\text{Han}) = 0.34$, and $\nu_{\text{Quartz with clay}} = 0.17$, for the data listed in Mavko et al. 2009. For the data given by Wang et al., (1998), Poisson’s ratios range between 0.20 and 0.32 for a variety of clay minerals.

From these numbers it is clear that quartz-rich lithologies will, on average, have a higher Young’s modulus and a lower Poisson’s ratio than clay-rich lithologies. A high brittleness index rock by a mineralogical definition (i.e. due to a high $V_{\text{Quartz}}$) will result in a high Young’s modulus $E$ and low Poisson’s ratio $\nu$ in the aggregate material. The definitions of high brittleness index using a mineralogical definition (Equations 1 and 2) and a definition by elastic properties are therefore equivalent.

To demonstrate that the definition of brittleness index by rock strength is also equivalent to the definition from elastic parameters and from a mineralogical description we use a linear Mohr-Coulomb failure criterion. We have already established that the definition of a brittleness index by the ratio of compressive strength to tensile strength implies a steeper slope of the shear failure envelope in a linear Mohr-Coulomb failure criterion. The slope $\mu$ of a linear Mohr-Coulomb failure envelope is related to the angle of internal friction $\varphi$ by $\mu = \tan^2\left(\frac{\varphi}{4} + \frac{\pi}{2}\right)$, a monotonously increasing function. Therefore a large $\mu$ also implies a large $\varphi$, and vice versa. Evidence from numerous studies (e.g. Plumb, 1994, Vernik et al., 1993 and Crawford et al., 2010) shows conclusively that rock with a large volume of quartz as part of the rock matrix (i.e. exhibiting a large brittleness index according to the mineralogical definition) display an increased friction angle $\varphi$ compared to clay rich rocks. For low porosity
sediments (say with porosities in the range of 0-15%) typical values for clean sandstone (i.e. very quartz rich rocks) observed friction angles range from 35 to 60 degrees. In grain-supported rocks, where the rock matrix contains a fair percentage of clay minerals (i.e. wackes), observed friction angles range from 30 to 40 degrees. For shale lithologies, comprising pre-dominantly clay minerals, observed friction angles range from 20 to 35 degrees.

We can now see that the definition of brittleness index via a mineralogical description, via a ratio of compressive strength and tensile strength, and via combinations of elastic parameters would all ascribe the highest brittleness index to clean sandstones, an intermediate brittleness index to mixed lithologies with grain support and a low brittleness index to rocks comprised purely of clay minerals.

**Beyond brittleness index: geomechanical models to assess hydraulic stimulation performance**

Understanding the hydraulic stimulation process is related to understanding failure of the rock at the wellbore wall and in the stimulated formation. Stated in the most basic form, rock failure occurs where stress overcomes the strength of the material. For the purposes of hydraulic stimulation, failure can further be differentiated into shear failure and tensile failure of either the intact rock matrix or along pre-existing planes of weakness, such as pre-existing fractures and faults or weak bedding planes. For tensile fracture opening with a fracture plane perpendicular to the minimum principal stress, the height growth of these fractures is governed by the stress contrast across the boundary between the stimulated layer and the adjacent formations. In the case of an increase in least principal stress across the boundary from the resource layer into adjacent formations, height growth is limited and lateral fracture propagation is encouraged. Vice versa, in the case of a decrease in minimum principal stress when crossing from a resource layer into an adjacent formation, a stimulated fracture will predominantly grow out-of-zone. In order to better understand the effect of hydraulic stimulation, we need to understand the stress state in conjunction with the elastic and strength properties of the rock mass.

*Fracture containment is governed by stress state*

For the sake of our argument, we use the poro-elastic strain equations (e.g. Blanton and Olsen, 1999), which describe horizontal total stresses in a horizontally layered Earth as functions of Young’s Modulus $E$, Poisson’s Ratio $v$, Pore Pressure $P_p$, Biot’s Constant $\alpha$, the total vertical stress magnitude $\sigma_v$, and the minimum and maximum horizontal tectonic strains $\varepsilon_{\text{hmin}}$ and $\varepsilon_{\text{hmax}}$.

$$\sigma_{\text{hmin}} = \frac{v}{1-v}(\sigma_v - \alpha P_p) + \frac{E}{1-v^2}\varepsilon_{\text{hmin}} + \frac{E v}{1-v^2}\sigma_{\text{hmax}} + \alpha P_p,$$

$$\sigma_{\text{hmax}} = \frac{v}{1-v}(\sigma_v - \alpha P_p) + \frac{E}{1-v^2}\sigma_{\text{hmax}} + \frac{E v}{1-v^2}\varepsilon_{\text{hmin}} + \alpha P_p.$$  

Eq. 4a  

Eq. 4b

These equations state that for a specific lithological column with elastic property and pore pressure variations with depth, the magnitude of the principal horizontal stresses is depends on the tectonic context, given by the tectonic strain parameters $\varepsilon_{\text{hmin}}$ and $\varepsilon_{\text{hmax}}$. In early case studies where extensive measurements were taken to derive minimum and maximum horizontal stresses at several points along a wellbore (e.g. Blanton and Olson, 1999, and Evans et al., 1989), it was clearly established that horizontal stresses don’t follow a simple gradient, but that there is variability of minimum and maximum horizontal stresses with lithology and elastic properties. The tectonic strain parameters $\varepsilon_{\text{hmin}}$ and $\varepsilon_{\text{hmax}}$ are in practice used as two calibration parameters such that calculated stresses from the poro-elastic stress equations correctly predict field observations. Field observations include (but are not limited to) minimum principal stresses inferred from mini-frac and extended leak-off tests, direct pressure measurements, inference of pore pressure from kicks and losses, caliper logs and image log interpretations of wellbore breakouts and drilling induced tensile fractures. Note, that we do not discuss the question of availability, sufficiency or quality of calibration data, necessary for this calibration procedure.

Note that the key mechanical properties in the poro-elastic strain equations are Young’s modulus ($E$) and Poisson’s ratio ($v$). Recall that a combination of high $E$ and low $v$ denotes a high brittleness index and therefore it is often implicitly assumed to be a desirable location to drill a well. The implicit assumption behind this line of reasoning is
that a high brittleness index rock will fail in a brittle manner, and a low brittleness index rock will fail in a ductile manner, whereby the brittle failure somehow generates a connected network of fractures in the resource layer, resulting in good drainage of the reservoir.

We are now going to demonstrate, using a simple thought experiment that this use of brittleness index is too simplistic and can be downright misleading. Using the same layered property model of a mudstone-sandstone-mudstone sequence characterized by representative values for $E$ and $\nu$, we will show that stimulated fractures can either be (i) predominantly contained in the high BI zone, (ii) not see the interface between high and low BI zones and (iii) predominantly grow out of the high BI zone. This is accomplished by looking for a set of tectonic strain parameters $\varepsilon_{\text{hmin}}$ and $\varepsilon_{\text{Hmax}}$ such that the variation in minimum horizontal stress $\sigma_{\text{hmin}}$ across the interfaces between mudstone-sandstone-mudstone sequence is such that (i) the sandstone interval shows a lower $\sigma_{\text{hmin}}$ than the adjacent strata, (ii) no stress contrast in $\sigma_{\text{hmin}}$ exists between the strata and (iii) the sandstone interval shows a higher $\sigma_{\text{hmin}}$ than the adjacent strata. It has to be noted, that the following analysis assumes that the minimum horizontal stress forms also the minimum principal stress.

First, we are determining the tectonic strains at which no stress contrast exists across an interface between two strata with different elastic properties. The simplified Earth model assumes two lithologies (e.g. sandstone and mudstone), each with constant $E$ and $\nu$.

The problem can be further simplified by assuming temporarily that both horizontal strains $\varepsilon_{\text{hmin}}$ and $\varepsilon_{\text{Hmax}}$ (and therefore minimum and maximum horizontal stresses within a formation) are equal. No stress contrast across the interface implies that $\sigma_{\text{hmin}(\text{sandstone})} = \sigma_{\text{hmin}(\text{mudstone})}$. Using the poro-elastic strain equations in each lithology and re-arranging terms shows that for tectonic strain values of:

$$\varepsilon_{\text{hmin}} = \varepsilon_{\text{Hmax}} = \frac{A}{B}$$

with

$$A = \left[\frac{v_{\text{sandstone}}}{1-v_{\text{sandstone}}} - \frac{v_{\text{mudstone}}}{1-v_{\text{mudstone}}}\right] \times \left(\sigma_v - \alpha P_p\right),$$

and

$$B = \left[\frac{E_{\text{mudstone}}(1+v_{\text{mudstone}})}{1-v_{\text{mudstone}}} - \frac{E_{\text{sandstone}}(1+v_{\text{sandstone}})}{1-v_{\text{sandstone}}}\right].$$

For the case of tectonic strain terms with different magnitudes, the condition for no stress contrast across the interface is:

$$A = C \varepsilon_{\text{hmin}} + D \varepsilon_{\text{Hmax}},$$

with

$$A = \left[\frac{v_{\text{sandstone}}}{1-v_{\text{sandstone}}} - \frac{v_{\text{mudstone}}}{1-v_{\text{mudstone}}}\right] \times \left(\sigma_v - \alpha P_p\right),$$

$$C = \left[\frac{E_{\text{mudstone}}}{1-v_{\text{mudstone}}} - \frac{E_{\text{sandstone}}}{1-v_{\text{sandstone}}}\right],$$

$$D = \left[\frac{E_{\text{mudstone}} v_{\text{mudstone}}}{1-v_{\text{mudstone}}} - \frac{E_{\text{sandstone}} v_{\text{sandstone}}}{1-v_{\text{sandstone}}}\right].$$

Note that the additional inequality of $\varepsilon_{\text{Hmax}} > \varepsilon_{\text{hmin}}$ must also be satisfied.

For the thought experiment, we use Equation 5 and assume a 10m thick tight sandstone reservoir at a depth of 2500m, bounded at the top and bottom by mudstone. The elastic properties for $E$ and $\nu$ are taken from Thiercelin and Plumb (1994), and serve as a guide for reasonable values. The tight sandstone and mudstone are characterized by $E_{\text{sandstone}} = 50\text{GPa}$ and $v_{\text{sandstone}} = 0.12$, and $E_{\text{mudstone}} = 27\text{GPa}$ and $v_{\text{mudstone}} = 0.27$, respectively. Vertical stress is calculated using a constant stress gradient of $\sigma/v = 0.024\text{MPa/m}$, pore pressure is calculated using a hydrostatic pore pressure gradient for brine of $Pp/z = 0.01174\text{MPa/m}$, and a Biot constant $\alpha = 1$. 

Evaluating the above equation (using consistent units for stress and Young’s modulus in MPa) the tectonic strain at which no stress contrast across the interface exists is $\epsilon_{hmin} = \epsilon_{Hmax} = 3.61e^{-4}$. Increasing the tectonic strain term causes the horizontal stress in the sandstone layer to become larger than the horizontal stress in the mudstone. Vice versa, decreasing the horizontal strain term causes a smaller minimum horizontal stress in the sandstone than in the mudstone (see Figure 1). Note that for all three cases, a vertical tensile fracture may develop during hydraulic stimulation, as the predicted stress state is in a normal faulting stress regime.

This difference in stress profiles shows that in a low strain environment (e.g. $\epsilon_{hmin} = \epsilon_{Hmax} = 3.2e^{-4}$) the mudstone with a low Young’s modulus and a high Poisson’s ratio exhibits increased horizontal stresses compared to the (tight) sandstone with high Young’s modulus and low Poisson’s ratio. When stimulating in this scenario within the sandstone, a tensile fracture would form in the sandstone, which may be deterred from further vertical growth by the increased horizontal stress in the mudstone (Figure 2a). This scenario would seem to support the commonly held opinion that the (high brittleness) sandstone will fracture and while the (low brittleness index) shale will not fail.

However, in the scenario of a slightly elevated tectonic strain (e.g. $\epsilon_{hmin} = \epsilon_{Hmax} = 4.0e^{-4}$), the same shale will exhibit decreased horizontal stresses compared to the sandstone. In this scenario, fractures will initially propagate through the sandstone, but then predominantly develop in the mudstone layers (Figure 2b).

![Figure 1: Profiles of minimum horizontal stress $\sigma_h$ and vertical stress $\sigma_v$ as a function of depth for a 10 m thick high brittleness index sandstone layer (high E, low v). Note that depending on the applied tectonic strain $\epsilon = \epsilon_{hmin} = \epsilon_{Hmax}$ the sandstone layer can display a lower minimum horizontal stress, the same minimum horizontal stress or a larger minimum horizontal stress than the bounding mudstone.](image-url)
Figure 2: Hydraulically stimulated fractures will show (a) predominant lateral growth, if the resource layer has a lower minimum principal stress than the bounding layer, and (b) predominant vertical growth (out of zone), if the resource layer has a higher minimum principal stress than the bounding layer.

In other words, a low strain environment implies high horizontal stress in low brittleness index units compared to high brittleness index units. In this scenario, hydraulic fractures may not be able to penetrate the low brittleness unit, and may stay contained in rocks displaying a high brittleness index. This scenario follows expectations of high brittleness index = unit is good for hydraulic stimulation = fractures are created and contained in quartz-rich units. On the other hand, in a high tectonic strain environment, the opposite can be true. Fractures may predominantly develop and propagate in rocks characterized by a low brittleness index. This results in a scenario of low brittleness index = good for hydraulic stimulation = fractures are generated and contained in resource mudstones and shales.

Note that in the high- and low-tectonic strain scenarios described above, fracture stimulation in a resource layer has to be analyzed in conjunction with the adjacent strata. The difference in elastic properties between different strata causes stress barriers between the strata and thus governs whether fractures stay contained in the resource layer, or grow out-of-zone.

Principles that improve upon the overly simplistic high brittleness index = good for hydraulic stimulation criterion for assessing suitability of a specific formation for hydraulic stimulation therefore need to incorporate the stress state and pore pressure in conjunction with elastic and strength properties. By investigating the relationship between mechanical properties and stresses, it is possible to make qualitative statements about fracture propagation that is indicative of the quality of a hydraulic fracture treatment in terms of fracture containment within a resource layer or out-of-zone growth. The stress state and pore pressure needs to be assessed via a calibrated geomechanical model. A property model of Young’s modulus and Poisson’s ratio is one necessary input for a successful analysis. However, knowledge of Young’s modulus and Poisson’s ratio in itself is not sufficient to understand the success for hydraulic stimulation, even in intact rock that does not contain fractures.
Re-activation of existing fractures or creation of new fractures?

A further factor that influences the suitability of a specific formation for stimulation by hydraulic fracturing is the presence or absence of naturally occurring fractures. Such natural fractures can form planes of weakness (if uncemented) and will, depending on their orientation, re-open during hydraulic stimulation before the host rock fails. In Figure 3, we show a sketch using a Mohr circle to depict the stress state together with two shear failure lines. The first (stippled) shear failure line indicates failure of the intact rock material, and the second (dotted) shear failure line indicates failure for a plane-of-weakness with zero cohesion. The two black dots indicate the point of failure of intact rock and initiation of slip for a pre-existing fracture (or fault), respectively. The orientation of pre-existing fractures with respect to the three principal stresses will govern the magnitude of effective normal stress and shear stress on the fracture/fault plane, and thereby the distance from the failure envelope. The main point of this sketch is to demonstrate that depending on fracture orientation and stress state, hydraulic stimulation will either (i) create new fractures in intact rock, (ii) re-activate pre-existing fractures, or (iii) induce slip along other planes of weakness (such as bedding planes). In some scenarios where pre-existing fractures are re-activated, this can reduce the pressure necessary for hydraulic stimulation compared to a scenario where new fractures need to be generated. Pre-existing fractures may also form an extensive network of hydraulically connected pathways and thereby aid production.

![Figure 3: Sketch depicting simultaneous initiation of shear failure of intact rock and slip along an existing fracture.](image)

Failure in shear or in tension?

A further consideration in understanding the hydraulic fracturing process is the mode of failure during propagation of hydraulically activated fractures. The subsurface stress state again plays a key role in determining the mode of failure. If the principal effective stresses \(\sigma_1', \sigma_2', \) and \(\sigma_3'\) are of similar magnitude an increase in pressure will likely result in a tensile failure. On the other hand, if there is a large difference in \(\sigma_1'\) and \(\sigma_3'\) the likely outcome of hydraulic stimulation is the creation of shear fractures. The necessary pressure increase for each failure mode is displayed in Figure 4 (adapted from Mildren et al., 2005).
Figure 4: Depending on the three effective principal stresses a pressure increase can result in formation fractures in shear, in tension, or in a mixed mode. The amount of pressure increase necessary to induce each failure mode is given by the equations in the figure.

Discussion and conclusions

Arguably, at the injection rates encountered during hydraulic stimulation, the ensuing strain rates governing the deformation process are such that most lithologies encountered in unconventional resource plays undergo brittle deformation. Brittle failure can thereby occur either in intact rock or along pre-existing planes-of-weakness such as fractures or weak bedding planes. Additionally, different failure modes can occur. The mode of failure (in tension, in shear or in a mixed mode) is governed by the stress state and strength of the material – and not by the elastic properties or the mineralogical composition of the stimulated rock. We have shown that commonly used brittleness indices, as tempting as the name may sound, are therefore, by themselves, not a suitable index to indicate the propensity of a formation to undergo brittle failure during hydraulic stimulation. Brittleness index is therefore also not a good measure or indicator whether a hydraulic network of connected fractures is created during hydraulic stimulation, which would enhance effective permeability.

The three main classes of brittleness index (being defined by the mineral composition, the rock elastic behaviour and a suitable combination of strength indicators) have all been shown to be, essentially, lithology indicators. High brittleness index is thereby typically associated with high quartz content or high dolomite content. In turn, the mineralogical composition affects both the elastic and strength properties. This knowledge can feed into a geomechanical model. Once suitably calibrated, a geomechanical model (comprising knowledge of elastic and strength properties of the intact rock and fracture, as well as pore pressure and stress state) brittle failure can be predicted. Using a thought experiment, we have shown that hydraulically stimulated fractures can predominantly occur in rocks with either a high or a low brittleness index. The implications of this thought experiment are that a target formation with identical reservoir properties can act both as a zone which contains hydraulic fractures (and can therefore be efficiently stimulated, resulting in good reservoir production) or a zone from which hydraulic fractures predominantly escape into adjacent rock formations (which can result in poor reservoir production).
References


